OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear System
Spring 1998
Midterm Exam #2

Name : ______________________________

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<u>Problem 1</u>:

Let

$$S = \left\{ x \in \Re^3 | x = \alpha \begin{bmatrix} 0.6 \\ 1.2 \\ 0.0 \end{bmatrix} + \beta \begin{bmatrix} 0.5 \\ 1.0 \\ 0.0 \end{bmatrix}, \alpha, \beta \in \Re \right\},$$

find the orthogonal complement space of S, $S^{\perp} (\subset \Re^3)$, and determine an orthonormal basis and dimension for $S^{\perp}$. For $x = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T (\in \Re^3)$. And find its direct sum representation (i.e., x_1 and x_2) of $x = x_1 \oplus x_2$, such that $x_1 \in S, x_2 \in S^{\perp}$.

Problem 2:

Let $V = F^3$, and let F be the field of rational polynomials. Determine the representation of $v = \begin{bmatrix} s+2 & \frac{1}{s} & -2 \end{bmatrix}^T$ in (V, F) with respect to the basis $\{v^1, v^2, v^3\}$, where $v^1 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T$, $v^2 = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, $v^3 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T$.

<u>Problem 3</u>:

Show that the determinant of the $m \times m$ matrix

$$\begin{bmatrix} s^{k_m} & -1 & 0 & \cdots & 0 & 0 \\ 0 & s^{k_{m-1}} & -1 & \cdots & 0 & 0 \\ 0 & 0 & s^{k_{m-2}} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & s^{k_2} & -1 \\ \beta_m(s) & \beta_{m-1}(s) & \beta_{m-2}(s) & \cdots & \beta_2(s) & s^{k_1} + \beta_1(s) \end{bmatrix}$$

is equal to

$$s^n + \beta_1(s)s^{n-k_1} + \beta_2(s)s^{n-k_1-k_2} + \cdots + \beta_m(s)$$

where $n = k_1 + k_2 + \cdots + k_m$ and $\beta_i(s)$ are arbitrary polynomials. (<u>hint</u>: proof by induction)

Problem 4:

Given is the system of first-order ordinary differential equation

$$\dot{x} = t^2 Ax ,$$

where $A \in \Re^{n \times n}$ and $t \in \Re$. Determine the state transition matrix $\Phi(t, t_0)$.

Problem 5:

Consider $x(k+1) = A(k)x(k)$. Define

$$\Phi(k,m) = A(k-1)A(k-2)\cdots A(m), \quad \text{for } k > m$$
$$\Phi(m,m) = I$$

Show that, given the initial state $x(m) = x_0$, the state at iteration k is given by $x(k) = \Phi(k,m)x_0$.

If A is independent of k, what is $\Phi(k,m)$?